Exam # ____

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DEPARTMENT OF PHYSICS

UNIVERSITY OF OREGON

Ph.D. Qualifying Exam

Part II: Quantum Mechanics and Statistical Physics

Room 115, Lawrence Hall

Thursday, September 25, 1997, 11 a.m. to 3 p.m.

• The examination papers are numbered in the upper right hand corner of each page. Sign your name and print it in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Your exam number is already on the question sheets. Be sure to place both the exam number and the question number on any pages you wish to have graded.

• There are six equally weighted questions, each beginning on a new page.

• Read all six questions before attempting any answers.

• Begin each answer on the same page as the question, but continue on blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start on a new page.

• If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

• Calculators may be used only for arithmetic. Calculators which can store equations or text are not allowed.

• Dictionaries may be used if they have been approved by the proctor before the examination begins.

• No other papers or books may be used.

• Please make sure you follow all instructions carefully. If you fail to follow instructions, or to stop working on the exam when the time is up, an appropriate number of points may be subtracted from your final score. When you have finished, remain in your seat and a proctor will collect your exam.

1. a) Explain, primarily in words, what is meant in quantum mechanics by the phase shifts of a spherical elastic scatterer, and why these are sufficient to specify its scattering properties completely.

b) For a spherically symmetric square well potential of depth V and radius R, what relationship between V and R produces a zero s-wave shift for low, but non-zero, energy ($E << \hbar^2/2mR^2$)? Estimate the smallest non-zero value of V for which this occurs (in terms of R, the mass m of the scattered particle, and fundamental constants).

c) What happens to the scattering cross-section if the s-wave phase shift goes to $\pi/2$ at low energy? Give a physical interpretation of the resulting dramatic effects.

2. Consider an atom modeled as a single valence electron moving in a central potential. Ignore spin. Suppose that the electron is in a state with angular momentum $l = 1$. Now suppose that the atom is placed in a weak external electric field leading to a potential

$$V_{\text{pert}}(\vec{r}) = A\,x^2 + B\,y^2 - (A+B)\,z^2 \quad ; \quad \vec{r} = (x, y, z) \quad .$$

Here coordinates are chosen such that the atomic nucleus is at the origin. Consider the effects of V_{pert} in perturbation theory to lowest order.

a) Show that the $l = 1$ level splits into three distinct levels, each with a wave function of the form

$$\psi(\vec{r}) = (\alpha x + \beta y + \gamma z)\, f(r) \quad ,$$

with $r = |\vec{r}|$, and $f(r)$ a function that is common to all three levels. Determine the constants α, β, and γ for each of the three levels.

hint: It is okay to use the above form of $\psi(\vec{r})$ as an ansatz.

b) Sketch the energy level diagrams, specifying the **relative** shifts ΔE in terms of the potential parameters A and B. (You do not need to determine the absolute shifts, i.e. calculating the shifts up to a common factor is okay.)

c) Compute the expectation value of the z-component of the angular momentum, L_z, for each of the three levels.

3. Use the variational method to study the energy levels of a particle in the 1-dimensional potential

$$V(x) = \begin{cases} \infty, & \text{if } x \leq 0 \quad ; \\ c\,x, & \text{if } x > 0 \quad . \end{cases}$$

Using $\psi_0(x) = \Theta(x)\; xe^{-ax}$ as a trial wave function, find a bound for the ground state energy E_0. Here $\Theta(x)$ is the step function, i.e. $\Theta(x) = 0$ for $x < 0$ and $\Theta(x) = 1$ for $x \geq 0$.

4. a) For a system at constant volume in equilibrium with a heat bath at temperature T, derive an expression for the mean square fluctuation of the internal energy in terms of the heat capacity C.

b) Suppose now that two samples, A and B, with heat capacities C_A and C_B, respectively, and previously in equilibrium at temperature T, are placed in thermal contact with one another but are otherwise thermally isolated. In terms of these heat capacities, derive an expression for the mean square fluctuation of the internal energy of one of the samples, after the combined system has reached equilibrium.

5. A harmonic oscillator with mass m and angular frequency ω is in equilibrium with a heat bath at absolute temperature T.

a) Calculate the partition function of the oscillator classically.

b) Calculate the partition function of the oscillator quantum mechanically.

c) Find the internal energy, the entropy, and the heat capacity of N such quantum mechanical oscillators as functions of the temperature.

d) Check that your result in part c) yields the correct classical limit for the internal energy and the heat capacity.

hint: $\int_{-\infty}^{\infty} dx\, e^{-x^2} = \sqrt{\pi}$.

6. The following data apply to the triple point of water:

Temperature: 0.01° C.

Pressure: 4.58 mm of Hg.

Specific volume of solid: $1.0907\,\mathrm{cm}^3/\mathrm{g}$.

Specific volume of liquid: $1.0001\,\mathrm{cm}^3/\mathrm{g}$.

Heat of fusion: 80 calories/g.

Heat of vaporization: 596 calories/g.

a) Sketch a P-T diagram for water which need not be to scale, but must be qualitatively correct.

b) Consider a point in the liquid phase at a temperature of -1° C. Now the pressure is slowly reduced to zero, while the temperature is kept constant. Describe any phase changes that occur, and calculate the pressures at which they occur, assuming that water vapor may be treated as an ideal gas.

hints: (1) 1 calorie = 4.184 Joules.
(2) $k_B = 1.38 \times 10^{-23}$ Joules/K.
(3) 760 mm of Hg correspond to $1\,\mathrm{atm} = 10^5\,\mathrm{N/m}^2$.