Exam # ____

Name Printed: ______________________

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DEPARTMENT OF PHYSICS

UNIVERSITY OF OREGON

Ph.D. Qualifying Exam

Part I: Mechanics and Electromagnetism

Room 115, Lawrence Hall

Wednesday, September 24, 1997, 11 a.m. to 3 p.m.

• The examination papers are numbered in the upper right hand corner of each page. Sign your name and print it in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Your exam number is already on the question sheets. Be sure to place both the exam number and the question number on any pages you wish to have graded.

• There are six equally weighted questions, each beginning on a new page.

• Read all six questions before attempting any answers.

• Begin each answer on the same page as the question, but continue on blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start on a new page.

• If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

• Calculators may be used only for arithmetic. Calculators which can store equations or text are not allowed.

• Dictionaries may be used if they have been approved by the proctor before the examination begins.

• No other papers or books may be used.

• Please make sure you follow all instructions carefully. If you fail to follow instructions, or to stop working on the exam when the time is up, an appropriate number of points may be subtracted from your final score. When you have finished, remain in your seat and a proctor will collect your exam.

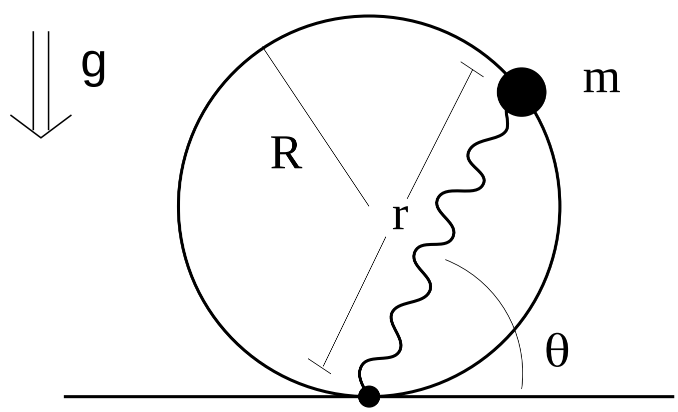
1. A bead of mass m slides without friction on a fixed vertical hoop of radius R. The bead moves under the combined action of gravity (acceleration g) and a spring attached to the bottom of the hoop. For simplicity, we **assume that the equilibrium length of the spring is zero**, so that the force due to the spring is $-kr$, where r is the instantaneous length of the spring. Assume that the bead can pass unhindered through the point at the bottom of the hoop.

a) Write down the Lagrangian and the Hamiltonian for the problem, taking θ as the canonical position variable.

b) Derive the general equations of motion for the problem.

c) If the bead is released at the top of the hoop with negligible speed, then how fast is it moving when it reaches the bottom of the hoop?

d) Find the frequency of oscillation if the bead is slightly displaced from the bottom of the hoop and then released.



2. Consider a pendulum consisting of a rigid thin rod with length L and negligible mass supporting a ball of mass M. The pendulum is immersed in a viscous medium which causes a frictional force F whose magnitude is proportional to the speed v of the ball: $F = -\mu v$. It swings in a vertical plane under the influence of gravity.

a) Derive the equations of motion of the pendulum, allowing for arbitrary angles of deflection θ from the vertical axis.

b) Express the second-order equation above in terms of two first-order equations

$$\dot{x} = f(x, y) \quad , \quad \dot{y} = g(x, y) \quad ,$$

using $x = \theta$ and $y = \dot{\theta}$, where the dot denotes d/dt. Fixed points (x^*, y^*) are defined by $f(x^*, y^*) = g(x^*, y^*) = 0$. Small deviations from each fixed point satisfy the matrix equation

$$\frac{d}{dt}\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} = \mathcal{A}\begin{pmatrix} \delta x \\ \delta y \end{pmatrix} \quad .$$

Determine the matrix $\mathcal{A}$ at each fixed point.

c) Find the eigenvalues of $\mathcal{A}$, and determine for each fixed point the critical value of the drag coefficient μ above which there is no oscillation.

3. A slowly moving antiproton is captured by a deuteron at rest, producing a neutron and a neutral pion:

$$\bar{p} + D \rightarrow n + \pi^0$$

The rest masses of the particles involved are $m_{\bar{p}} \approx m_n \approx m_D/2 \approx 939\,\mathrm{MeV}$, $m_{\pi^0} \approx 135\,\mathrm{MeV}$. Find the total energy of the π^0 emitted.

4. A plane electromagnetic (EM) wave is incident on a free particle of charge q and mass m. The EM wave causes the particle to oscillate and hence to radiate. This interaction can be considered as a scattering of EM radiation with cross-section $\sigma_T =$ (power radiated)/(incident flux). Assume low-energy incident EM waves, so that one may treat the interaction nonrelativistically. Using Larmor's radiation formula

$$P = \frac{1}{4\pi\epsilon_0}\frac{2}{3c^3}\left(\frac{d^2\mu}{dt^2}\right)^2 \quad ,$$

where μ is the **electric** dipole moment and P is the power radiated, show that

$$\sigma_T = \frac{1}{(4\pi\epsilon_0)^2}\frac{8\pi}{3}\left(\frac{q^2}{mc^2}\right)^2 \quad .$$

Evaluate σ_T for an electron.

5. In a purely classical model, we consider a dielectric medium as a collection of uncoupled, classical harmonic oscillators. Assume that each oscillator consists of an electron connected to a fixed ion by a harmonic spring with frequency ω_0.

a) Write down the equation of motion for the electron when a monochromatic electric field with frequency ω is applied.

b) For an electron density n, calculate the electric polarization and the dielectric constant $\epsilon(\omega)$.

c) For free electrons, $\omega_0 = 0$. For a free electron gas, at what frequency is $\epsilon = 0$? What is the physical significance of this frequency?

6. Consider a point charge q located at a distance d from the center of a grounded conducting sphere of radius $a < d$. Use spherical coordinates and locate the charge on the z-axis.

a) Find the potential outside the sphere.

b) Derive an expression for the induced surface charge density on the sphere.

c) Expand the potential in a power series in $1/r$ for $r >> d$ and keep the first two terms. What is the significance of these two terms? Find the electric field to the same order.